

## ANALYSIS OF THE SPECTRAL SIGNATURE OF BREAKING WAVES

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**Introduction:** Wave breaking plays an important role in air-sea interaction processes that affect the world's climate and the cycle of greenhouse gases in Earth's atmosphere. Over many decades, wave breaking is assumed to be spectrally broadbanded. Investigation of source function balance of short ocean waves, however, leads to the conclusion that dissipation function displays a quasi-singularity behavior. In other words, the spectral property of wave breaking is localized in wavenumber, with a strong signature at wave components near 1 m or shorter, depending on the degree of swell influence. The result is consistent with observed properties of radar sea spikes and frequency characteristics of sea surface sound generated by bubble clouds.

**Analysis:** For wind-generated waves, the dissipation source term remains the most difficult to formulate. Reference 1 describes an approach to establish the dissipation function in the equilibrium region of gravity waves where nonlinear wave-wave interaction is much weaker than wind input and breaking dissipation. Theoretical and experimental studies of the energy transfer from wind to waves have led to the wind input parameterization function,

$$S_w(k) = m\sigma (u_*/c)^2 N(k), \quad (1)$$

where  $k$  is wavenumber,  $m \approx 0.04$ ,  $\sigma$  is intrinsic frequency,  $u_*$  is wind friction velocity,  $c$  is wave phase speed, and  $N$  is wave action density. Expressed in terms of dimensionless spectral density,  $B = k^4 = \sigma N/g$ , where  $g$  is gravitational acceleration, the input and dissipation functions can be written as  $S_w(k) = m(u_*/c)^2 gk^{-4}B(k)$  and  $D = gk^{-4}f(B)$ . Equating  $S_w$  and  $D$  leads to the implicit equation for the unknown dissipation relation  $f(B)$ ,

$$\frac{\partial B}{\partial \xi} = m \left[ \frac{\partial}{\partial B} \left( \frac{f(B)}{B} \right) \right]^{-1}, \quad (2)$$

where  $\xi = (u_*/c)^2$ . It is concluded that if the variation of  $B$  with  $u_*/c$  can be established reliably,  $f(B)$  can be derived from Ref. 2.

**Application to Field Data:** By applying a free drifting operation to minimize Doppler frequency effect, spectra of short waves between 0.02 and 6 m (0.5 to 12 Hz) are collected from the open ocean.<sup>2</sup> The environmental conditions cover a wide range of wind speeds and sea states (Table 1). The data are further divided into wind seas (Subset 1) and mixed seas (Subset 2). Figure 4 shows scatter plots of  $B(u_*/c)$ ; the data can be represented by a power law function,  $B(u_*/c) = A_0(u_*/c)^{a_0} = A_0\xi^{a_0/2}$ . The coefficient and exponent,  $A_0$  and  $a_0$ , are given in Figs. 5(a) and 5(b). For a power law dissipation relation,<sup>1</sup>  $f(B) = A_d B^{a_d}$ ,  $A_d$  and  $a_d$  can be calculated by

$$a_d = 1 + \frac{2}{a_0}, \quad (3a)$$

$$A_d = mA_0^{1-a_d}. \quad (3b)$$

The results are given in Figs. 5(c) and 5(d).

**Discussions and Conclusions:** The presence of a localized region in wavenumber where the spectral density is only weakly dependent on the forcing wind condition (Fig. 5(b)) suggests that wave growth is quenched by strong dissipation in that region. For Subset 1 (wind waves),  $a_0$  decreases to less than 0.5, with a minimum of 0.22, for  $\lambda$  between 0.16 and 2.1 m. Correspondingly, the dissipation function in this region shows a very nonlinear dependence on  $B$ , as strong as  $B^{10}$  and approaches a delta-function (Fig. 5(c)-(d)). These wavelengths are sufficiently long that viscous dissipation or parasitic capillary generation as an energy sink of gravity waves may be neglected. Wave breaking as the major dissipation mechanism can be reasonably assumed. These results indicate that the length scale of breaking wind waves is localized in the range of about 0.16 to 2.1 m. The presence of swell broadens the breaking scales toward shorter waves (breaker size ranges between 0.2 and 0.9 m), possibly through wave-current interaction.

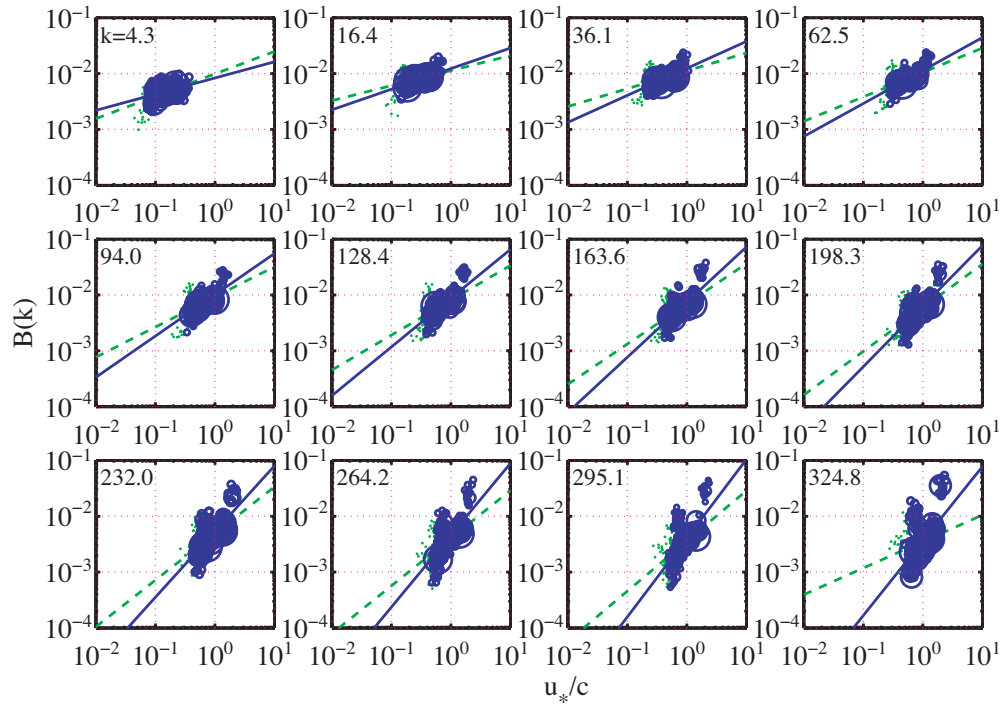
[Sponsored by ONR]

### References

- <sup>1</sup> O.M. Phillips, "On the Response of Short Ocean Wave Components at a Fixed Wavenumber to Ocean Current Variations," *J. Phys. Oceanogr.* **14**, 1425-1433 (1984).
- <sup>2</sup> P.A. Hwang and D.W. Wang, "An Empirical Investigation of Source Term Balance of Small Scale Surface Waves," *Geophys. Res. Lett.* **31**, L15301, doi:10.1029/2004GL20080.

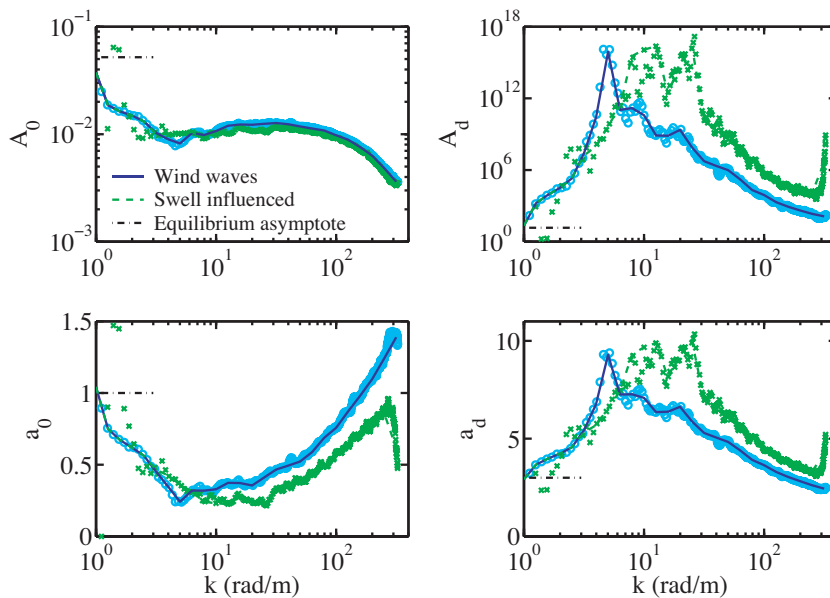
Table 1 — Summary of Experimental Conditions

	No. of Spectra	$U_{10}$ (m/s)	$u_*$ (m/s)	$H_s$ (m)	$T_a$ (s)
Subset 1	291	3.6 - 14.2	0.11 - 0.55	0.21 - 2.84	1.93 - 6.33
Subset 2	106	2.6 - 10.2	0.063 - 0.40	0.21 - 0.75	2.07 - 4.68



**FIGURE 4**

Scatter plots of  $B(u_*/c)$  for different spectral wave components. The wavenumber is marked at the top left of each panel. Circles and solid curves are for Subset 1, dots and dashed curves are for Subset 2 (from Ref. 2).



**FIGURE 5**

(a)  $A_0$ , and (b)  $a_0$  of the empirical power law function  $B(u_*/c)$ , and (c)  $A_d$ , and (d)  $a_d$  of the dissipation power law function  $f(B)$ . Note that the range of  $A_d$  covers 18 orders of magnitude. For wind seas, the coefficient increases about four orders of magnitude between  $4 < k < 6$  (breaker size ranges between 1 and 1.5 m). In the presence of swell, the breaking scale moves to higher wavenumber,  $7 < k < 30$  rad/m (breaker size ranges between 0.2 and 0.9 m). (From Ref. 2.)